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NOTES

ON THE COMPATIBILITY OF A NEW EXPRESSION FOR GROSS CONVERSION EFFICIENCY WITH THE VON BERTALANFFY GROWTH EQUATION[1]

Gross food conversion efficiency (K_1) is defined by

$$K_1 = \text{growth increment/food ingested} \quad (1)$$

$$= \frac{dW}{dt}/I$$

where I is the ingestion rate (Ivlev 1939; Ricker 1966); data from feeding experiments are usually fit to an allometric model of the form

$$K_1 = c\,W^{\alpha} \quad (2)$$

where W is the body weight, and c and α are empirical constants which, however, have the disadvantage of always predicting values of $K_1 > 0$, although the fish and other aquatic animals to which the model is meant to apply usually experience size constraints and hence must reach a value of W where $K_1 = 0$. It is therefore preferable to choose a functional form for K_1 which falls to zero as W approaches W_∞. Furthermore, recent analysis of feeding studies of a number of fish species indicates that K_1 can approach arbitrarily close to unity for the smallest fishes, which suggests the alternate equation

$$K_1 = 1 - (W/W_\infty)^{\beta} \quad (3)$$

where W_∞ is the weight at which $K_1 = 0$, and β is an empirical constant estimated from the slope of

$$\log(1 - K_1) = \beta \log W - \beta \log W_\infty \quad (4)$$

(Pauly 1986).

In this note we show that Equation (3) is compatible with the von Bertalanffy growth function (VBGF), both in its standard (von Bertalanffy 1938) and generalized forms (Richards 1959; Pauly 1981), which is not true of Equation (2).

We assume that the ingestion rate (I) can be expressed as an allometric expression of weight of the form

$$I = HW^{d}, \quad (5)$$

where H and d are empirical constants. From Equation (1) we then obtain for the growth rate

$$dW/dt = K_1\,HW^{d} \quad (6)$$

which combined with Equation (3) gives

$$dW/dt = (1 - (W/W_\infty)^{\beta})\,HW^{d} \quad (7)$$

and hence

$$dW/dt = HW^{d} - kW^{m} \quad (8)$$

where $m = d + \beta$ and $k = H/W_\infty^{\beta}$. Equation (8) is the differential form of the VBGF, and can be integrated for various values of the constants m and d. Setting $d = 2/3$ and $m = 1$ (i.e., $\beta = 1/3$) yields the "normal" VBGF for weight,

$$W_t = W_\infty\,(1 - e^{-K(t-t_0)})^3 \quad (9)$$

where $K = k/3$, while if $m = 1$ and $0 < d < 1$ we get the generalized VBGF sensu Pauly (1981),

$$W_t = W_\infty\,(1 - e^{-KD(t-t_0)})^{3/D} \quad (10)$$

where $D = 3(1 - d)$. This second form is probably more useful as it allows for the exponent of the allometric relationship linking ingestion and weight (Equation (5)) to take wider range of values, as needed to fit various data sets and/or to mimic various models in the literature (see, e.g., Paloheimo and Dickie 1966 or Ursin et al. 1985).

The compatibility shown here between the recently proposed Equation (3) expressing K_1 as a function of fish weight and the VBGF is encouraging, as it supports the method suggested by Pauly (1986) for combining these two equations when estimating the food consumption of fish populations and leads to a mathematically consistent approach for the analysis of feeding and growth data.

[1]ICLARM Contribution No. 316.

LITERATURE CITED

BERTALANFFY, L. VON.
1938. A quantitative theory of organic growth (Inquiries on

growth laws II). Hum. Biol. 10:181-213.
IVLEV, V. S.
1939. Balance of energy in carps. [In Russ.] Zool. Zh. 18: 303-318.
PALOHEIMO, J. E., AND L. M. DICKIE.
1966. Food and growth of fishes III. Relations among food, body size and growth efficiency. J. Fish. Res. Board Can. 23:1209-1248.
PAULY, D.
1981. The relationship between gill surface area and growth performance in fish: a generalization of von Bertalanffy's theory of growth. Meeresforschung 28:251-282.
1986. A simple method for estimating the food consumption of fish populations from growth data and food conversion experiments. Fish. Bull., U.S. 84:827-840.
RICKER, W. E.
1966. "The biological productivity of waters" by V. S. Ivlev. J. Fish. Res. Board Can. 23:1717-1759. [Transl.]
RICHARDS, F. J.
1959. A flexible growth function for empirical use. J. Exp. Bot. 10:290-300.
URSIN, E., M. PENNINGTON, E. B. COHEN, AND M. D. GROSSLEIN.
1985. Stomach evacuation rates of Atlantic cod (*Gadus morhua*) estimated from stomach contents and growth rates. Dana 5:65-80.

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