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A Method for Estimating the Parameters of a Seasonally Oscillating Growth Curve from Growth Increment Data*

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Introduction

Various studies summarized in Longhurst and Pauly (1987) have demonstrated that the growth of fishes and aquatic invertebrates oscillates seasonally, even in the tropics. Such growth oscillation, which can be detected in most sets of length-at-age data, can also be detected in most sets of growth increment data, e.g. from tagging-recapture data, even in coral reef fishes (Pauly and Ingles 1982). To be able to do this, however, one must use a model structured such that growth oscillations will be picked up if they occur in the growth increment data at hand. Such models have been presented by Pauly and Ingles (1981), Pauly (1984) and Appeldoorn (1987), who adapted for use with growth increment data the seasonally oscillating growth model of Pauly and Gaschütz (1979). Appeldoorn's model, as reformulated by Somers (1980), is a modified version of the von Bertalanffy growth function (VBGF) of the form:

$$L_{t2} = L_\infty \left[1 - (1 - \frac{L_{t1}}{L_\infty}) \exp \left[-K (t_2 - t_1) + \frac{CK}{2\pi} (\sin 2\pi (t_1 - t_s) - \sin 2\pi (t_2 - t_s)) \right] \right] \quad ...1)$$

where L_{t1} and L_{t2} are growth increments, with "tagging" dates t_1 and "recovery" date t_2, respectively, when K and L_∞ are defined as in the standard VBGF and where C and t_s express the amplitude of the oscillation and the position of the first sinusoid growth oscillation with reference to t=0, respectively.

Appeldoorn (1987) used a non-linear technique - the MICROSIMPLEX routine of Schnute (1982) - to fit equation (1) to data. We present here a linearized version of equation (1) which can be implemented when routines for non-linear fitting are not available.

Model development and fitting

Factoring out $\sin 2\pi t_s$ in equation (1) by expanding the seasonality term, gives

$$\sin 2\pi (t_1 - t_s) = \sin 2\pi t_1 \cos 2\pi t_s - \cos 2\pi t_1 \sin 2\pi t_s \quad ...\ 2a)$$

$$\sin 2\pi (t_2 - t_s) = \sin 2\pi t_2 \cos 2\pi t_s - \cos 2\pi t_2 \sin 2\pi t_s \quad ...\ 2b)$$

Thus, for a given L_∞, equation (1) can be turned into a multiple linear regression of the form:

$$Y = b_1 X_1 + b_2 X_2 + b_3 X_3 \quad ...\ 3)$$

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$$\text{where} \quad Y = \ln \frac{(L_\infty - L_2)}{(L_\infty - L_1)}$$

$$b_1 = -K \quad ; \quad X_1 = (t_2 - t_1)$$

$$b_2 = \frac{CK\cos 2\pi t_s}{2\pi} \quad ; \quad X_2 = \sin 2\pi t_1 - \sin 2\pi t_2$$

$$\text{and} \quad b_3 = \frac{CK\sin 2\pi t_s}{2\pi} \quad ; \quad X_3 = \cos 2\pi t_2 - \cos 2\pi t_1$$

The parameter K can be directly estimated from b_1 while t_s is estimated from

$$t_s = \tan^{-1}(b_3/b_2)/2\pi$$

Then, C can be estimated from either b_2 or b_3.

One feature of equation (3) is that although it has the form of a multiple regression, it may be difficult to estimate its parameters using a statistical software package, because many such packages do not allow for a multiple linear regression lacking in constant term. A box is, therefore, provided below which gives an explicit method for estimating the parameters of equation (3) using the sum of squares and products as defined by the least square method for estimating bi's from a set of observations. This is expressed in matrix form as:

$Y = XB$ where,

$$Y = \begin{bmatrix} Y_1 \\ \cdot \\ \cdot \\ \cdot \\ Y_2 \end{bmatrix} \quad X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ X_{n1} & X_{n2} & X_{n3} \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

where the b coefficients are estimated using

$$B = (X'X)^{-1} \; X'Y$$

(see Box for explicit form of the content of these matrices).

The goodness of fit of the model to the data can be estimated from $R^2 = 1 - \Sigma(Y-Y)^2/\Sigma(Y-Y)^2$, i.e., from the sum of square of the residuals of the model over the sum of squares of the deviations from the mean.

Estimation of the parameters of equation (1), as applied to a given set of growth increment data, would thus imply:

i) calculate values of X_1, X_2 and X_3,

ii) estimate parameters b_1, b_2, and b_3 of equation (3), given a "seed" value of L_∞ (e.g., a value 5% larger than the largest L_2 value in the data set),

iii) compute the value of R^2 of the model in (ii),

iv) return to (ii) with a new estimate of L_∞, estimate B_1, B_2 and B_3, and K, C and t_s, and compare new R^2 value with previous R^2 value. Stop when R^2 cannot be further improved and accept the last estimate of L_∞ and its associated estimates of K, C and t_s.

Table 1 provides a set of growth increments, illustrating the type of data required for this method.

Table 1. Growth increment data derived by tagging a coral reef fish, the ocean surgeonfish *Acanthurus bahianus* from the Virgin Islands[a].

Increment No.	Total Length (cm) L_1	L_2	Dates tagging (t_1)	recovery (t_2)
1	9.7	10.2	20/04/59	12/06/59
2	10.5	10.9	29/05/59	01/07/59
3	10.9	11.8	13/11/59	29/02/60
4	11.1	12.0	12/06/59	22/09/59
5	12.4	15.5	13/08/59	11/05/60
6	12.8	13.6	12/06/59	30/07/59
7	14.0	14.3	23/04/59	15/06/59
8	16.1	16.4	28/04/59	10/07/59
9	16.3	16.5	13/04/59	15/06/59
10	17.0	17.2	07/04/59	22/07/59
11	17.7	18.0	22/04/59	11/08/59

[a] Extracted from Table 3 in Randall (1962); data included pertain only to fish which grew at least 2 mm while at large; this accounts for small measurement errors and case of no-growth due to tagging wounds (see also Table 4.5. Fig. 4.7 in Pauly 1984).

Results and discussion

Table 2 gives the estimates of L_∞, K, C and t_s obtained by applying our method to the data in Table 1, along with estimates obtained using other methods also included in the ELEFAN V program of the Compleat ELEFAN Software package of Gayanilo et al. (1988).

Table 2. Growth parameters of ocean surgeonfish *Acanthurus bahianus* in the Virgin Islands, as estimated from tagging-recapture data in Table 1 using various models of which some do not consider seasonality in growth[a]

L_∞ (cm)	K (year^{-1})	C	t_s	WP	ssr[b]	ϕ' [c]	Method
20.34	0.432	>0.51[d]	n.a.	Mar/Apr[d]	1.154	2.25	Gulland and Holt (1959)
20.68	0.485	n.a.	n.a.	n.a.	0.942	2.32	Fabens (1965)
19.25	0.532	n.a.	n.a.	n.a.	1.112	2.29	Munro (1982)
29.03	0.225	0.55	0.82	April	0.739	2.28	Eq. (1) and MICROSIMPLEX
26.90	0.265	0.67	0.83	April	0.783	2.28	Equation (3) and Box

[a] In such cases, the entries under C, t_s and WP are not applicable (n.a.)
[b] sum of squares of the residuals using the VBGF model
[c] $\phi' = \log_{10}K + 2\log_{10}L_\infty$
[d] As estimated using the residuals of the Gulland and Holt plot and an original routine implemented in the Compleat ELEFAN package (see Gayanilo et al. 1988).

As might be seen, the method proposed here generates with our test data a residual sum of squares that is only slightly higher than using the non-linear MICROSIMPLEX routine, but which is much lower than using the other three methods considered here. Also, the estimates of C, t_s and WP obtained by the MICROSIMPLEX and equation (3) are very close to each other. The estimates of L_∞ and K differ slightly, but the differences compensate each other, i.e., the two values of the growth performance index (Pauly and Munro 1984) are the same.

Based on this, and other applications not documented here, we conclude that the method presented here, of linearizing and fitting equation (1) to data, will be useful in cases when a non-linear routine is not available and especially, to provide good "seed" values for a non-linear routine.

$$b_1 = \frac{\Sigma X_1Y\{\Sigma X_2^2\Sigma X_3^2-(\Sigma X_2X_3)^2\}+\Sigma X_2Y\{\Sigma X_1X_3\Sigma X_2X_3-\Sigma X_3^2\Sigma X_1X_2\}+\Sigma X_3Y\{\Sigma X_1X_2\Sigma X_2X_3-\Sigma X_2^2\Sigma X_1X_3\}}{|\,X'X\,|}$$

$$b_2 = \frac{\Sigma X_1Y\{\Sigma X_1X_3\Sigma X_2X_3-\Sigma X_3^2\Sigma X_1X_2\}+\Sigma X_2Y\{\Sigma X_1^2\Sigma X_3^2-(\Sigma X_1X_3)^2\}+\Sigma X_3Y\{\Sigma X_1X_2\Sigma X_1X_3-\Sigma X_1^2\Sigma X_2X_3\}}{|\,X'X\,|}$$

$$b_3 = \frac{\Sigma X_1Y\{\Sigma X_1X_2\Sigma X_2X_3-\Sigma X_2^2\Sigma X_1X_3\}+\Sigma X_2Y\{\Sigma X_1X_2\Sigma X_1X_3-\Sigma X_1^2\Sigma X_2X_3\}+\Sigma X_3Y\{\Sigma X_1^2\Sigma X_2^2-(\Sigma X_1X_2)^2\}}{|\,X'X\,|}$$

and

$$|\,X'X\,| = \Sigma X_1^2\Sigma X_2^2\Sigma X_3^2-\Sigma X_1^2(\Sigma X_2X_3)^2+\Sigma X_2^2(\Sigma X_1X_3)^2-\Sigma X_3^2(\Sigma X_1X_2)^2+2\Sigma X_1X_2\Sigma X_2X_3\Sigma X_1X_3$$

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