Pauly, D. 1989. A theory of fishing for a two-dimensional world. *Fishbyte, Newsletter of the Network of Tropical Fisheries Scientists* 7(3): 28-31.

A Theory of Fishing for a Two-Dimensional World

DANIEL PAULY
International Center for Living
Aquatic Resources Management (ICLARM)
MC P.O. Box 1501
Makati, Metro Manila
Philippines

Introduction

In 1884, Edwin A. Abbott, a schoolmaster with a passion for theology and literature, published - pseudonymously - a little book titled "Flatland: a romance of many dimensions" in which he explored some of the implications of living in a two-dimensional world.

He described a world in which there is Left and Right, and Back and Forth, but no Up and Down, and dealt with issues such as the climate and houses, the inhabitants (especially the women, who, as opposed to the round males, were pointed and hence, had to be treated with great respect), the problems of color recognition and other issues illustrating the differences between Flatland and a three-dimensional world such as ours.

Mainly, however, he dealt with moral and theological issues - this was the thing to do in the Victorian era. So, the emphasis of "Flatland" was devoted to the conflicts between the local clergy (who were "Administrators of all business, art and science"), and those Flatlanders, philosophers and mystics, who were spreading seditious notions, such as "third dimension", "cube" or "upward".

A.K. Dewdney published in 1984 "The Planiverse: computer contacts with a two-dimensional world", in which the idea of Flatland was carried further (see also Gardner 1980). Interestingly and characteristically for the age we live in, it was not the moral/theological aspects which received Dewdney's attention, but the technological puzzles: how to build two-dimensional machines (locks, steam engines, electrical motors), how to organize social interactions (sex, sports, warfare), or how basic physical principles would operate (sound and light propagation, expansion of the planiverse, etc.). Not being a Victorian author, I shall continue on Dewdney's path and present some first elements of a theory of fishing for a two-dimensional universe in case a reader establishes contact with a two-dimensional world and is asked to provide advice on the management of the fisheries there.

The Ardean Background

Following Dewdney (1984), I shall refer to a disc-shaped planet called "Arde", peopled by "Ardeans", and in which there is "up" and "down" and "forward" and "backward", but no "left" and "right" (which is easier to conceive than Abbott's tabletop "Flatland"). Fig. 1 shows two Ardeans on a small fishing vessel, carrying several fishing traps. This, or any other representation of Ardean life and technology must be conceived as lacking any depth whatsoever (e.g., one cannot differentiate between a flatfish and a roundfish). Dimensional constraints preclude the existence of all kinds of nets (think a bit, you'll see why), and hence, the Ardeans use for fishing, besides traps, only hooks and dynamite, the last of these to the great chagrin of Ardean conservationists.

On Arde as on Earth, the key factors which determine fish production are growth, mortality, stock size and recruitment (Russel 1931). We now discuss these factors and their mathematical representation as they apply to Ardean fish and fisheries.

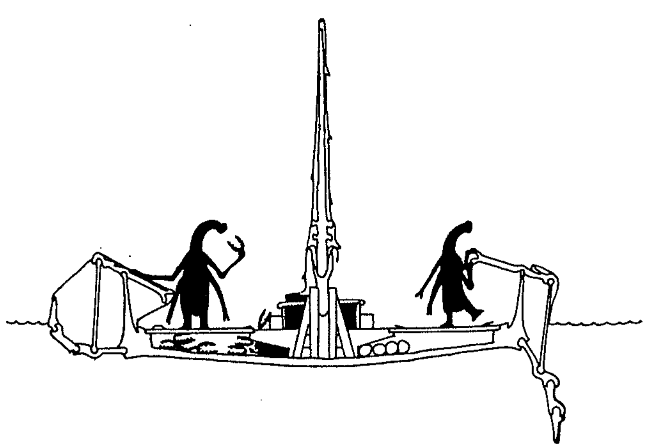


Fig. 1. Two Ardeans on a fishing craft; note fishing traps under the craft's deck and in the hand of the Ardean at the fore of the vessel (reproduced without permission from Dewdney 1984).

Growth of Ardean Fishes

The von Bertalanffy equation (Bertalanffy 1938) can be used to model the growth of Ardean fishes, i.e.,

$$L_t = L_\infty (1-e^{-K(t-t_o)}) \quad \text{...1)}$$

where L_t is the length at age t,
L_∞ is the length of very, very old fishes
K is a constant of dimension 1/t expressing the rate at which L_∞ is approached, and
t_o is the length at "age" zero (see Beverton and Holt 1957, Pauly 1984).

Ardean fishes have no weight. Their dimension relevant to fishery catches is surface area, and this grows according to

$$S_t = S_\infty (1-e^{-K(t-t_o)})^b \quad \text{...2)}$$

where S_t is the surface area at age t
S_∞ is the surface area of very, very old fishes
K is a constant of dimension 1/t expressing the rate at which S_∞ is approached
t_o is as defined above, and
b is the exponent of a length-surface relationship of the form

$$S = aL^b \quad \text{...3)}$$

When b = 2, equation (3) is called isometric and represents the general case; when b <> 2, the equation is either positively (b > 2) or negatively (b < 2) allometric.

In Ardean fishes as in ours, overall metabolic rate (and hence, growth) is limited by the relative "size" of the respiratory organs. On Earth, this "size" has the dimension of a surface, i.e., the gill surface area (Pauly 1981). On Arde, this size has the dimension of length, i.e., it is that part of the fishes periphery that is thin enough to be used for gas exchange. The growth of this line occurs with a positive allometry, as shown on Fig. 2 [note that many Ardean fishey biologists, as on Earth, argue rather strongly that the growth of their fishes has nothing to do with respiration, see e.g., Weatherley and Gill 1987].

Fig. 3A illustrates the growth in surface area of an Ardean fish; note the inflection point occurring at a very low age and surface area.

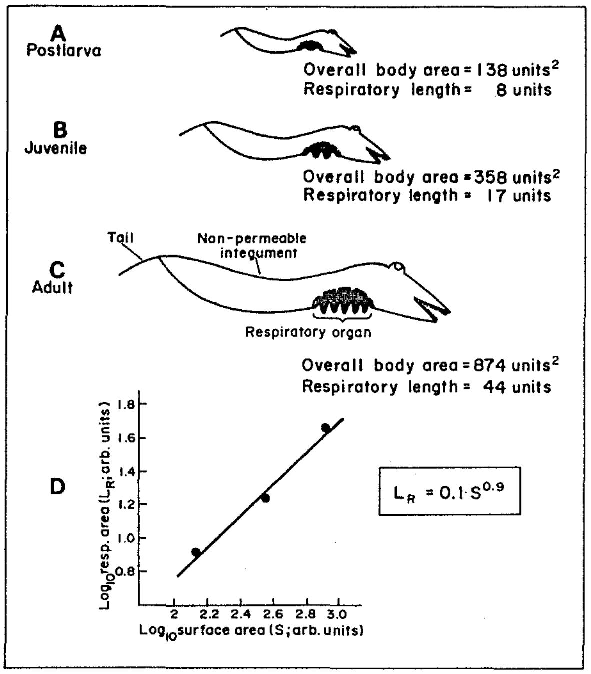


Fig. 2. Schematic representation (using arbitrary units) of the allometric growth of the respiratory apparatus of the Ardean fishes; the crucial dimension for respiratory performance here is length, i.e., respiratory length (corresponding to respiratory surface in our fishes). Note that this length increases, from A to C, faster than the rest of the periphery of the fish (i.e., the length of the non-respiratory integument), but that this length still cannot grow as fast as the fish's surface (hence the exponent of the lengths/surface relationship in D is < 1). This is what causes growth rate to gradually decline, down to zero at S_∞.

Mortality (Natural, Fishing and Total)

The mortality of Ardean fishes can be modelled, as that of our fishes, by

$$N_{t2} = N_{t1} \cdot e^{-Z(t_2 - t_1)} \qquad ...4)$$

where N_{t1} and N_{t2} are population sizes at two successive times, e = 2.7182..., the base of the "natural" or Ardean logarithms, and Z is the instantaneous rate of total mortality. We obviously also have Z = M + F, where M and F are natural and fishing mortalities, respectively.

As on Earth, and especially in the North Sea, the value of M in Ardean fishes is generally set equal to 0.2. Due to transmission problems, however, I have been unable to check whether this estimate, communicated to me by a noted Ardean scientist, was expressed on a daily, weekly or seasonal basis.

The method used by Ardean fishery scientists to estimate F is quite precise: they close all fisheries for a while. When they are satisfied that all fishing has stopped, they declare that $F = 0 \pm 0$ (see Fishbyte 6(1):4-5, 1988 for a comment on the need to provide confidence intervals about estimates).

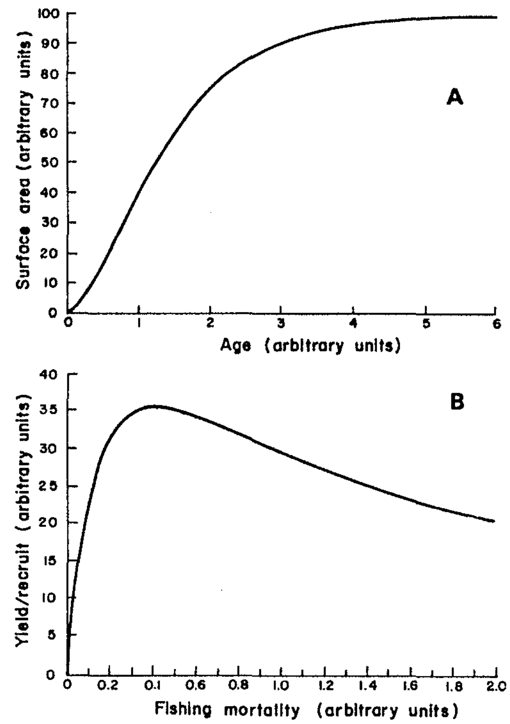


Fig. 3. Aspects of the population dynamics of Ardean fishes.
A. Growth in surface of a fish with parameters S_∞ = 100, K = 0.2 and t_o = -0.5 (arbitrary units)
B. Yield per recruit of that same fish with M = 0.2, and $t_r = t_c$ = 1 (arbitary units).

Recruitment of Ardean Fishes

Recruitment of Ardean fishes tends to occur in one pulse, at the beginning of the winter (i.e., rainy) season.

Ardean fishery scientists also work on the "recruitment" problem, i.e., on the fact that recruitment seem to vary more or less independently of spawning stock sizes.

Their work is not at all easy: they can't use nets for sampling the eggs (remember why?) nor hooks and lines to sample young larvae (yolk egg stages don't bite!). On the other hand, female Ardean fish are far less fecund than ours (the space in their ovaries grows only with the square of their length, as opposed to the cube as in our fishes) and hence, there are less eggs and larvae floating about and dying or getting lost and adding variability to the recruitment process. [Perhaps, there is more to recruitment studies on Arde, but there also were problems with the transmission of this part.]

Stock Size and Yield Per Recruitment

The overall surface area of a cohort of Ardean fishes can be obtained, obviously, by multiplying equation (2) and (4), i.e., combining the functions for S_t and N_t. When b = 2, this gives, after expansion of equation (2)

$$S_t \cdot N_t = S_\infty (1-2e^{-k(t-to)} + e^{-2k(t-to)}) \cdot N_{to}\, e^{-z(t-to)} \qquad ...5)$$

The rate of capture (in units of surface area) is

$$\frac{dY_s}{dt} = F \cdot S_t \cdot N_t \qquad \text{...6)}$$

and in the spirit of Beverton and Holt (1957), the catch (Y_s) is obtained by integration, i.e.,

$$Y_S = F \cdot \int_{t_c}^{t_{max}} S_t \cdot N_t \, dt \qquad \text{...7)}$$

where t_c is the age at first capture, and t_{max} is the longevity of the fishes. Integrating between t_c and ∞ (i.e., assuming $t_{max} = \infty$), and reexpressing on a per-recruit basis gives

$$\frac{Y_s}{R} = F \cdot e^{-Mr_2} \cdot S_\infty \left[\frac{1}{Z} - \frac{2e^{-Kr_1}}{Z+K} + \frac{e^{-2Kr_1}}{Z+2K} \right]$$

where r_1 is t_c-t_0
r_2 is t_c-t_r, and
t_r is the age at recruitment. Fig. 3B shows an Ardean yield-per-recruit curve.

Discussion

I am as puzzled as the reader about how this paper could ever be written, let alone be accepted for publication. Thus, providing a "Discussion" could be construed as adding insult to injury. However, the deed is done, so let's discuss it, if only briefly.

Constructing imaginary worlds with exaggerated features is an old literary ploy; Jonathan Swift, the creator of Lilliput and Brobdingnag, immediately comes to mind, and he used this device to castigate 18th Century England. The Flatland of Rev. Abbot was, similarly, a satire of Victorian England.

Dewdney's book is also a satire, but an involuntary one. Its author blissfully constructs one (two-dimensional) contraption after the other, while failing to realize that our time is precisely one in which we scientists, caught in a limited world of our own, keep constructing neat little gadgets which are then taken from us, and dropped onto people.

Dewdney also makes - equally involuntarily - an interesting statement about fishery science: he totally ignores it. He had asked some of his friends - astronomers, physicists, chemists, biologists - to help him "flesh out" Arde and hence, his book covers the basic sciences as comprehensively as it does technology. However, in spite of the importance of fish and shellfish to the Ardean diet, fishery science had been neglected. It will be for us, members of the NTFS, to correct for this omission: let's apply our science to the two-dimensional world of Arde (but let's not fall into the trap of dealing only with the technological aspects either)! Any suggestions?

Acknowledgment

My thanks to Ms. Mina Soriano for letting herself be coerced into helping me prepare this contribution.

References

Abbott, E.A. 1884. Flatland: a romance of many dimensions. Reprint of 7th (1952) Revised Edition, Dover Publications, New York, NY, 103 p.

Bertalanffy, L. von. 1938. A quantitative theory of organic growth (Inquiries on growth laws II) Hum. Biol. 10(2): 181-213.

Beverton, R.J.H. and S.J. Holt. 1957. On the dynamics of exploited fish populations. Fish. Invest. Ser. II, 19, 533 p.

Dewdney, A.K. 1984. The Planiverse: computer contacts with a two-dimensional world. Picador/Pan Books, London.

Gardner, M. 1980. The pleasure of doing science and technology in the Planiverse. Sci. Am. 243(1): 14-20.

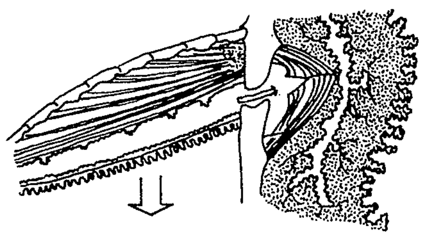
Pauly, D. 1981. The relationships between gill surface area and growth performance in fish: a generalization of von Bertalanffy's theory of growth. Meeresforsch. 28(4): 251-282.

Russel, E.S. 1931. Some theoretical considerations on the overfishing problem. J. Cons. CIEM 6(1): 3-20.

Weatherley, A.H. and H.S. Gill. 1987. The biology of fish growth. Academic Press, London, 443 p.

Swimming and Breathing

The swimming fin [of Ardean fishes] consists of a series of jointed bones on its leading edge, a complicated series of muscles next to this, a tissue layer of unknown purpose, a long, narrow cavity, and a gill on the trailing edge. [For Ardean fishes, to]



swim is to breathe. When the fin is inflated, it stands out from the body and the long muscles immediately contract, forcing the fin rapidly toward the rear. At this point in the swimming cycle, the gill at the rear of the fin is most actively [involved in gas exchange with] the surrounding water. Simultaneously, the two body bones are disarticulated and the contracting space inside the fin pushes its fluid through the opening and against the portal muscle. The zipper of this muscle pumps all the fluid into the 'pulmonary' cavity even as the bones come once again together and the short muscles of the fin begin their contraction phase. The fin begins to curl as it is drawn forward alongside the body. Meanwhile, having just inflated the pulmonary chamber, the portal muscle now deflates it, creating a bubble against the still articulated bones. When these are forced apart once more, the portal muscle contracts violently, inflating the fin again.

From Dewdney (1984), with small modifications [in square brackets].