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A note on Ursin's formulae for the estimation of natural mortality in fish stocks.

by

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Introduction

The present note is a by-product of a study of the interrelationships between growth parameters and natural mortality coefficients in various fish stocks (PAULY, 1978).

The growth parameters used here are L_∞, W_∞ and K of the Von Bertalanffy Growth Formula (VBGF) which has the form

$$L_t = L_\infty (1 - e^{-K(t - t_o)}) \text{ for length} \quad \ldots.1)$$

and

$$W_t = W_\infty (1 - e^{-K(t - t_o)})^3 \text{ for weight.} \quad \ldots.2)$$

For reasons to be discussed elsewhere, the parameter K of the VBGF is referred to as "stress factor". Also, the word "size" (abbreviated S) will be used here instead of L_∞ and/or W_∞ whereever possible.

URSIN (1967) attempted to demonstrate the existence of a relationship between size and natural mortality in fishes, using natural mortality estimates and values of S selected from Table I in BEVERTON & HOLT (1959). The plot of M against log"W"$_\infty$ in his Appendix Fig. 12 (p. 2433) indeed suggests the existence of such a relationship. From this plot and subsequent computations, URSIN (1967) derived two rules of thumb for the estimation of mortality, namely:

a) "for species of the size order $W = 10^3$g, we have $M = W^{-1/3}$" and

b) "when the weight is multiplied by 10 the natural mortality is halved".

Both rules of thumb have been used since by several authors.

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No attempt will be made here to deal with the theoretical basis for the derivation of these rules of thumb. It appears, however, that they both provide quite inaccurate estimates of M when applied to any fish stocks outside of temperate waters. It appears also that there is no direct relationship between M and size in the data used by URSIN (1967) for the derivation of his rules. This is demonstrated in this note.

Material, Method and Results

The L_∞, K and M data selected from BEVERTON & HOLT (1959) by URSIN (1967) are given here in Table I. Note that URSIN (1967) used for the conversion from length to weight the same condition factor (0.9) for all fishes.

The correlation coefficients between $\log L_\infty$, $\log "W"_\infty$, logK and logM are given in Table IIA. All are highly significant, thus seemingly demonstrating the existence of a relationship, among other things, between logM and $\log "W"_\infty$. The relationship between logM and logK, however, is even closer, and it would therefore seem to be of interest to investigate what becomes of the relationship between logM and $\log "W"_\infty$ after the effect of logK has been removed

This can be investigated by calculating the partial correlation coefficient, that is, the correlation coefficient expressing the degree of association between two variables after the effect of a third variable has been eliminated. Interesting partial correlation coefficients between the variables S, K and M would be, in our case, $r_{KM \cdot S}$, which expresses the degree of association between logK and logM after the effect of logS has been removed, and $r_{SM \cdot K}$, which expresses the degree of association between logS and logM after the effect of logK has been removed.
The formulae are:

$$r_{KM \cdot S} = \frac{r_{SK} - r_{SM} \cdot r_{KM}}{\sqrt{(1 - r^2_{SM}) \cdot (1 - r^2_{KM})}} = 0.488 \qquad \ldots.3)$$

and

$$r_{SM\cdot K} = \frac{r_{SM} - r_{SK} \cdot r_{KM}}{\sqrt{(1 - r^2_{SK}) \cdot (1 - r^2_{KM})}} = -0.270 \qquad ....4)$$

where, with thirty (30) degrees of freedom (n - 3), $r_{KM\cdot S}$ is significant at the 99% level, while $r_{SM\cdot K}$ is not significant. (Table II A) This lack of relationship between logM and logS can also be shown when plotting logM against logS and logK in a multiple regression of the form:

$$\log M = a + b \log S + c \log K \qquad ....5)$$

The parameter values obtained for both logM plotted against $\log L_\infty$, logK and for logM against $\log "W"_\infty$, logK are given in Table II B. While the multiple correlation coefficient (R = 0.766 in both cases) is highly significant, it appears that the value of b, that is, the slope linking size with natural mortality, is not significantly $\neq 0$ (Table II B). What we have here is the fact that the stress factor K affects both S and M and that, therefore, a direct link between S and M is suggested which in fact does not exist - or at least cannot be demonstrated solely on the basis of the data in Table I.

Discussion

The present note does not aim at disproving the existence in fishes of a direct relationship between natural mortality and size. Indeed such a relationship does exist, as can be demonstrated by using a larger body of empirical mortality and size data, and by eliminating the effects of associated values of K and of environmental temperature (PAULY, 1978).

The point here is that the relationships proposed by URSIN (1967) are misleading because they directly relate M and size, although the material (Table I) shows that, in fact, there is a direct relationship only between M and K, the apparent relationship between M and size being due to the effect of K on size. The results, in this case, are that the real effect of size on mortality- which is quite small (See PAULY, 1978) - becomes overestimated, and that the proposed rules of thumb become grossly inaccurate in fishes with high values of K, for example, in tropical fishes.

SACHS (1974, pp. 352-353) gives a discussion of an analogous case, in the field of medicine, where an assumed relationship between varicose veins and other circulatory disorders whose correlated occurrence had led early authors to assume the existence of a "Status varicosus" which was demonstrated by WAGNER (1955) to be an artifice caused by the direct relationships between age and varicose veins on the one hand, and age and other circulatory disorders on the other.

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Table I. Data of BEVERTON & HOLT (1959) used by URSIN (1967, Appendix, Table VII, p. 2431).

No.	Species	L_∞	"W"$_\infty$	K	M
1	Clupea harengus	30.	243.	0.38	0.25
2	Clupea harengus	21.	82.3	0.65	0.78
3	Clupea pallasii	23.	111.	0.29	0.56
4	Sardinops caerula	26.	158.	0.39	0.15
5	Gadus morhua	132.	20700.	0.2	0.2
6	Gadus minutus ♂	20.	72.	0.42	1.1
7	Gadus minutus ♀	24.	124.	0.40	0.9
8	Gadus virens	107.	11025.	0.19	0.15
9	Merluccius merluccius ♂	44.	767.	0.13	0.6
10	Merluccius merluccius ♀	60.	1944.	0.10	0.5
11	Pleuronectes platessa ♂	45.	820.	0.15	0.22
12	Pleuronectes platessa ♀	70.	3087.	0.08	0.12
13	Pseudopleuronectes americanus	44.	767.	0.4	0.3
14	Solea vulgaris	39.	534.	0.4	0.25
15	Acipenser fulvescens	178.	50758.	0.05	0.01
16	A. medirostris/ A. transmontanus	300.	243000.	0.06	0.03
17	Blennius pholis	17.	44.2	0.30	0.9
18	Callionymus lyra ♂	25.	141.	0.43	0.96
19	Callionymus lyra ♀	17.5	48.2	0.55	0.86
20	Cottus gobio ♂	7.2	3.36	0.7	1.1
21	Cottus gobio ♀	7.3	3.50	0.4	0.9
22	Cottus gobio ♂	6.5	2.47	0.90	0.90
23	Cottus gobio ♀	6.5	2.47	0.50	0.80
24	Phoxinus phoxinus	9.	6.56	0.55	1.1
25	Gasterosteus aculeatus	6.7	2.71	0.64	0.9
26	Pungitius pungitius	4.3	0.72	1.6^{+}	1.1
27	Cynoscion macdonaldi	128.	18874.	0.3	0.3
28	Perca fluviatilis	30.	243.	0.20	0.29
29	Perca fluviatilis	34.	354.	0.13	0.16
30	Stizostedion canadensis	40.	576.	0.14	0.44
31	Dasyatis akajei ♀	150.	30375.	0.1	0.45^{++}
32	Pneumatophorus diego	40.	576.	0.4	0.9
33	Neothunnus macropterus	190.	61731.	0.5	0.8

$^{+}$1.94 in URSIN (1967) $^{++}$0.4 in URSIN (1967)

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Table II A. Correlation coefficients.

Two-variable coefficients		Partial correlation coefficients	
r_{SK}	-0.748	$r_{KM \cdot S}$	0.488
r_{KM}	0.745	$r_{SM \cdot K}$	-0.270*
r_{SM}	-0.677		

* $r_{SM \cdot K}$ not significant at 95%. ($r_{SM \cdot K}$ Critical Value = 0.349)

Table II B. Multiple Regression of M on S and K.

$$\log M = 0.3940 - 0.2632 \log L_{\infty} + 0.7248 \log K \qquad ....6)$$

$$\log M = 0.2145 - 0.0873 \log "W"_{\infty} + 0.7247 \log K \qquad ....7)$$

Equations 6 and 7: Critical value:(99%): R = 0.766 0.514. DF = 30, n = 33.

Standard deviation for log estimates of M = 0.3107.

95% confidence intervals for partial regression coefficients:

Equation 6) $b = -0.2632 \pm 0.3538$; $c = 0.7248 \pm 0.4848$.

Equation 7) $b = -0.0878 \pm 0.1170$; $c = 0.7248 \pm 0.4848$.